An Aggregate Flow Model for Air Traffic Management

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Traditionally, models used in air traffic control and flow management are based on simulating the trajectories of individual aircraft. This approach results in models with a large number of states, which are intrinsically susceptible to errors and difficult for designing and implementing optimal strategies for traffic flow management. This paper outlines an innovative approach for the development of linear time variant dynamic traffic flow system models based on historical data about the behavior of air traffic. The resulting low-order, linear, robust models can be used both for the analysis and synthesis of traffic flow management techniques for current and future systems.

I.   Introduction

Demand for air transportation has seen a six-fold increase in the past 30 years, and estimates call for a strong average annual growth rate of 4.7% during the next 20 years.[1] This increase in demand will put a further strain on the airports and sectors within the National Airspace System (NAS). The United States Congress has recognized the impact of increased demand and has established a Joint Planning and Development Office for creating and developing a Next Generation Air Transportation System to transform NAS operations. There are more than 40,000 commercial flights operated in the U. S. airspace alone on a typical day at the present time. In order to ensure that this traffic moves smoothly and efficiently in the presence of disruptions caused by convective weather and airport conditions, innovative modeling and design methods are needed in traffic flow management (TFM).

Today, air traffic flow prediction is done by propagating the trajectories of the proposed flights forward in time and using them to count the number of aircraft in a region of the airspace. The Center TRACON Automation System (CTAS) and the Future Automation Concepts Evaluation Tool (FACET) use this physics-based modeling approach for demand forecasting. The accuracy of these predictions is impacted by departure and weather uncertainties.[2-3] These trajectory-based models predict the behavior of the NAS adequately for short durations of up to 20 minutes. With the short prediction accuracy, it is difficult, if not impossible, to make sound strategic decisions on air traffic management.

A strategic TFM decision may involve rerouting all aircraft originating from the west coast, heading to airports on the east coast, to deal with anticipated stormy weather conditions near Chicago over the next 4 hours. Strategic TFM is a hierarchical system consisting of large number of states, and operating over time scales extending from a few hours to 24 hours. As shown in Fig. 1, the airspace in the United States is divided into 20 Centers in the continental United States plus one each in Alaska and Hawaii. The flow relationship between neighboring Centers is

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shown via links in Fig. 2. For example, the figure shows that Kansas City Center (ZKC) receives and sends traffic to the Minneapolis Center (ZMP). Proper mixes of strategic and tactical flow controls initiated by the System Command Center and the 22 Control Centers accomplish TFM in the U. S. Some of the frequently used flow restrictions include ground stop, ground delay, metering (miles-in-trail and time-based) and rerouting. Dispatchers and air traffic coordinators at airlines respond to these flow control actions by rescheduling and canceling flights, thus changing flow patterns.

Since strategic TFM requires control of flows of aircraft rather than individual aircraft, an aggregate model of traffic flow that does not use trajectories of individual aircraft is desirable. Strategic TFM can be substantially improved by the development of simpler, but more accurate models that allow the exploitation of different analysis and synthesis techniques from Systems Theory. Motivated by this objective, this paper describes a direct method for computing an aggregate model of air traffic flows from historical data.

In the recent literature, Ref. 4 describes a method for spatially aggregating air traffic for generating models of air traffic flow in an interconnected network of one-dimensional control volumes. An alternative approach to modeling air traffic flows using flow relationship between adjacent Centers is described in Ref. 5. The Linear Dynamic Systems Model (LDSM) in Ref. 5 is built by counting the number of aircraft entering a Center from an adjacent Center, number of aircraft leaving a Center for a neighboring Center and the numbers of aircraft landing and taking off within a Center. Input to this model consists of the number of departures. Results presented in Ref. 5, assuming that departures follow a Poisson distribution, show that the resulting numbers of aircraft in the Centers also fit a Poisson distribution. The main limitation of the results in Ref. 5 is that modeling departures from Poisson distributions (albeit a different one for each major hub airport) ignores the fact that departure counts vary significantly during the day as banks of aircraft arrive and depart major hub airports. Aircraft counts in the Centers, forecast by LDSM, can be improved significantly by accounting for the nominal departure rates as a function of time and augmenting them by modeling departure uncertainty about these nominal rates.

In this paper, the basic time-invariant LDSM proposed in Ref. 5 has been extended to a time-varying one. Instead of a single state transition matrix, several state transition matrices (one for each hour) were used to cover the entire prediction period. State transition matrices were computed using historical air traffic data. The resulting model was then driven by average departure rates, also derived from historical air traffic data, to predict aircraft counts in the 23 airspace regions. These 23 regions consisted of 20 Centers in the continental United States, one each covering Hawaii and Alaska, and one for the international airspace.

Uncertainty bounds around these nominal predictions were then obtained using the standard state covariance propagation model driven by the covariance of departure counts. Day-to-day variations about the average departure counts are assumed to be zero-mean Gaussian random variables. Four days of traffic data were used for computing both, the average departure counts and the variances about these average counts. Results are presented for another day of traffic data (other than the four days used in LDSM) to show that these counts lie within the confines of the mean aircraft counts predicted by the LDSM and uncertainty bounds generated by the covariance propagation technique. The main strength of the LDSM described here is that all the analytical tools available for analysis of linear dynamic systems can be applied to this model.

Fig. 1 Twenty Centers in continental U. S. airspace.
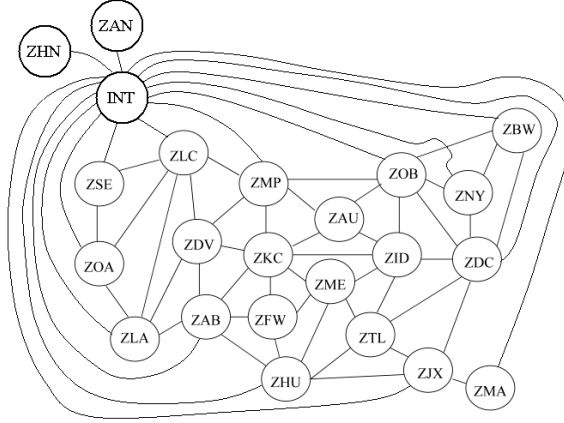
Fig. 2 Twenty-three Center network model.

LDSM is described in Sec. II. Section III describes the data from multiple days used for constructing and evaluating the model. Section IV describes the modeling of the state transition matrix using flow transition probabilities while Sec. V describes the model for the departures. In Sec. VI, the uncertainty bounds generated using the model are provided. Finally, concluding remarks are made in Sec. VII.

II.   Linear Dynamic System Model

A linear dynamic systems model for the air traffic in the NAS is developed in this section. This model can be used for predicting traffic count, which is the number of aircraft in a given Center, in the 22 Centers in the United States and one international region. The resulting traffic count forecast, which is a measure of future demand, can then be balanced against the available capacity using traffic flow management.

The number of arrivals (landings) and the number of aircraft leaving a Center in an interval of time, ΔT, are assumed to be proportional to the number of aircraft in the Center at the beginning of the interval. Following the notation in Fig. 3 and using the principle of conservation of flow (analogous to the principle of mass balance in a control volume) in a Center, the number of aircraft in Center i at the next instant of time, $k+1$, can be related to the number of aircraft in the Center at the current instant of time, k, via the difference in the number of aircraft that came into the Center and the number of aircraft that left the Center as follows.

$$x_i(k+1) = x_i(k) - \sum_{j=1}^{N} \beta_{ij} x_i(k) + \sum_{\substack{j=1 \\ j \neq i}}^{N} \beta_{ji} x_j(k) + d_i(k) \qquad (1)$$

The fractions β_{ij} and β_{ji} are obtained as transition probabilities in Ref. 5. The departures within Center i are denoted by $d_i(k)$. Observe that the model in Eq. (1) is driven solely by departures. All the other flow terms including the number of aircraft landing at a given instant of time are modeled as fractions of the numbers of aircraft in the Centers, which are the states of the model. If a database of such models is constructed using historical traffic data from days that characterize both the usual and unusual traffic flow conditions (for example, caused by weather conditions on the airport surface and enroute) in the NAS, the model that best represents the expected condition on a given day can be chosen for predicting that day's traffic.

For the purpose of modeling, the departures can be split into two components- a deterministic one and a stochastic one. The deterministic portion of the departures $u_i(k)$ can be computed from filed flight plans and from historical departure data. For example, $u_i(k)$ can be set to the average departure count derived from historical data. The stochastic component of the departures, $w_i(k)$, can be modeled by assuming a suitable distribution, such as a Gaussian or a Poisson distribution. In such a model, $w_i(k)$, which represents the expected variation around the deterministic component, can be obtained either from historical data or from the knowledge of that day's expected uncertainty.
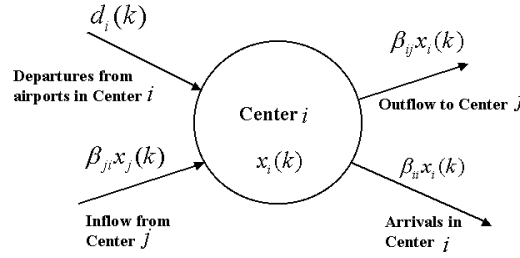
Fig. 3 The components of aircraft flow contributing to the traffic count in a given Center.

The discrete system in Eq. (1) can be rewritten in the standard State Space notation as:

$$\mathbf{x}(k+1) = \mathbf{A}(k)\mathbf{x}(k) + \mathbf{B}(k)\mathbf{u}(k) + \mathbf{C}(k)\mathbf{w}(k) \tag{2}$$

where,
- k denotes the time instant defined by $k\Delta T$, with ΔT being the sampling interval. In the earlier work in Ref. 5, it has been shown that a 10-minute sampling interval accurately approximates Center aircraft count. This sampling interval has also been used for generating the results presented in this paper;
- $\mathbf{x}(k) = [x_1(k),...x_N(k)]$ is the state vector with the number of aircraft in the Centers at time k as its elements;
- $\mathbf{u}(k) = [u_1(k),...u_N(k)]$ is the control vector with the number of aircraft departing (taking off) from the Centers as its elements;
- $\mathbf{w}(k) = [w_1(k),...w_N(k)]$ is a vector for modeling departure uncertainties;
- $\mathbf{A}(k)$ is the state transition matrix that contains the information of how flights transition from one Center to the other Center.

The elements of the state transition matrix A are given by:

$$a_{ij} = \beta_{ij};$$
$$i \neq j; i = 1,\cdots,N; j = 1,\cdots,N \tag{3}$$

where, $N = 23$ is the number of Centers including one for the international region. The off-diagonal terms $a_{ij}(k)$ represent the fraction of aircraft transitioning from Center i to the Center j at time k. This quantity can be calculated from historical data and will be shown to be slowly varying over time.

The diagonal terms can be calculated as:

$$a_{ii} = 1 - \sum_{j=1}^{N} \beta_{ij} \qquad (4)$$

These terms represent the fraction of the aircraft that remained in the Center i during the kth time step.

Although, a special case of the above formulation with $\mathbf{B} = 0$ and $\mathbf{C} = \mathbf{I}$ with $\mathbf{w}(k)$ being a Poisson random variable were used in Ref. 5, the general model with $\mathbf{B}$ permits analysis, such as the sensitivity of the traffic flow to variations in the departure rates. For the remainder of this paper, $\mathbf{B} = \mathbf{I}$ and $\mathbf{C} = \mathbf{I}$ are assumed, where $\mathbf{I}$ is the identity matrix.

III.   Data Used

Behavior of the LDSM was studied using real traffic data for four consecutive days from 6 May 2003 through 9 May 2003. Traffic data consisting of tracks and flight plans were recorded at one-minute intervals from the data feed provided by the Enhanced Traffic Management System (ETMS).[6] The four days of recorded data were processed using the Future ATM Concepts Evaluation Tool (FACET).[7] At each instant of time during the 24-hour period, numbers of aircraft in the Center, coming into the Center from a neighboring Center, leaving the Center for a neighboring Center, landing in the Center and taking off in the Center were counted for each of the 23 Centers (22 in U. S. + international). These counts were then used for creating the state transition matrices, **A,** for each day. The LDSM with these state transition matrices were subsequently used for generating time histories of traffic count in the Centers.

Traffic count results for the Atlanta Center (ZTL), Los Angeles Center (ZLA) and the New York Center (ZNY) as a function of Coordinated Universal Time (UTC), obtained using the LDSM driven by 6 May 2003 departures, are shown in Fig. 4. Note that the transition matrices used, one for each hour, were also created using May 6 recorded traffic data. As expected, these Centers have two basic modes of operation based on the time of the day. There is relatively little activity during the nighttime hours and a lot of activity during the daytime hours. Traffic increases rapidly during the local morning hours and then settles at a relatively high level for most of the day until the local nighttime hours. Observe from Fig. 4 that this basic pattern is offset between certain Centers, reflecting the differences between their local time zones.
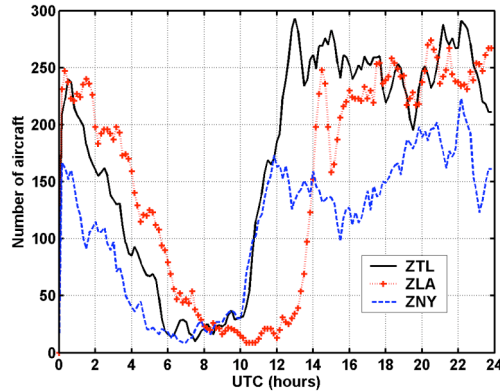
Fig. 4 Traffic count on 6 May 2003 in selected Centers in the U. S. across a 24-hour period.

Subsequent sections describe how the data were used for examining the properties of LDSM in forecasting nominal traffic count and uncertainty bounds about it.

IV.   Flow Matrix Modeling

This section examines two aspects of the LDSM: 1) the impact on modeling errors of maintaining state transition matrices constant over an aggregation intervals of ΔT (for example, one hour) during the 24-hour period and 2) the effect of using state transition matrices based on historical data for modeling today's traffic counts.

In Sec. II, the state transition matrix **A** was constructed using fractions of aircraft crossing Center boundaries. Although the number of aircraft does change from day-to-day (see Fig. 5), the benefit of using fractions is that they do not change very drastically. Fig. 5 shows the general behavior of Atlanta Center (ZTL) traffic across multiple days. Observe that the traffic patterns are quite similar but the number of aircraft at any given time of day does change from day-to-day depending on the number of departures.

Examination of the state transition matrix **A** shows that it is diagonally dominant with a large fraction of the aircraft staying within each Center between time steps k and $k+1$. For larger aggregation intervals, the diagonal dominance is diminished, with a large fraction of the aircraft leaving the Center. The off-diagonal terms increase because more aircraft arrive in the Center during the larger aggregation time interval.
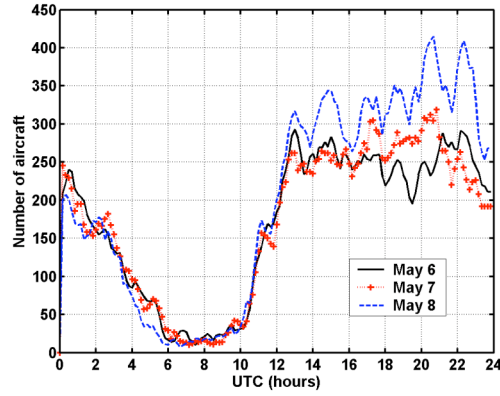

Fig. 5 Atlanta Center traffic across multiple days.

The slow moving nature of the underlying dynamics of flow and the diagonally dominant nature of **A** suggest that traffic count in the Centers can be modeled via a slowly varying **A** matrix. For example, Fig. 6 shows the effect of using **A** matrices averaged over one through 24-hour intervals in modeling traffic count in the Atlanta Center (ZTL) using 6 May 2003 data. Note that although state transition matrices are averaged over long time intervals, the LDSM is updated at every integration time step (for example, 10 minutes) for generating the time histories.

In order to study the impact of state transition matrix aggregation interval on the modeling errors (differences between the actual counts and those generated by LDSM), traffic counts were generated in the 23 Center airspaces using the state transition matrix averaged over several different aggregation intervals. Thus, for one-hour aggregation interval, 23 state transition matrices were used over the span of 24 hours, while, for 24-hour aggregation interval, a single state transition matrix was used. In each of these nine instances (1-hour though 24-hour aggregation), modeling errors were computed for the 24-hour day by taking the difference between the aircraft counts predicted by LDSM and the actual aircraft counts, determined from recorded data, at each discrete time step.
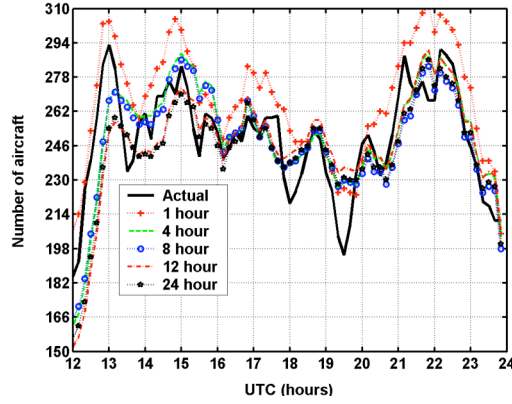
Fig. 6 Predicted aircraft counts in ZTL with u updated at every time step, but A aggregated over 1- hour through 24-hour time intervals using 6 May 2003 data.

Figure 7 shows the normalized mean of the modeling errors as a function of the aggregation interval for the Atlanta Center using 7 May 2003 data. Crosses on the graph show the computed data points. The modeling errors are normalized by expressing them as a percentage of the mean actual aircraft count. Normalized mean of the modeling errors is defined as:

$$\mu_N = \left(\frac{\mu_e}{\mu}\right)100 \tag{5}$$

where, μ_e is the mean of the modeling errors and μ is the mean of the actual aircraft counts. Note that modeling error is defined as the difference between the LDSM generated traffic counts and the actual traffic counts at each discrete time step. For 7 May 2003 data, the mean traffic in the Atlanta Center was 175 aircraft (see Fig. 5 for May 7 traffic count time history). Figure 7 shows that the normalized mean of the modeling errors, which represents the bias, is fairly small and does not change substantially with the aggregation interval.

The normalized standard deviation for the same data is also shown in Fig. 7. Normalized standard deviation of the modeling errors is defined as:

$$\sigma_N = \left(\frac{\sigma_e}{\mu}\right)100 \tag{6}$$

where σ_e is the standard deviation of the modeling errors and μ is the mean of the actual aircraft count, which for Atlanta Center on 7 May 2003 was 175. Observe from Fig. 7 that the standard deviation of the modeling errors grows with increasing aggregation interval.

The mean and standard deviation trends in the Atlanta Center on 6 May 2003 and 9 May 2003 were found to be very similar to those in Fig. 7. The trends were different for 8 May 2003 data in that the mean and the standard deviation of the modeling errors monotonically increased with increasing aggregation interval. For the 24-hour aggregation interval, the normalized mean and normalized standard deviation of the errors were found to be -13.78 (model-based counts less than actual counts) and 19.71, respectively, with the mean being 204 aircraft. Closer examination of the actual traffic data for 8 May 2003 revealed that Atlanta Center experienced a significantly greater traffic variation as a function of time of day compared to the three other days.
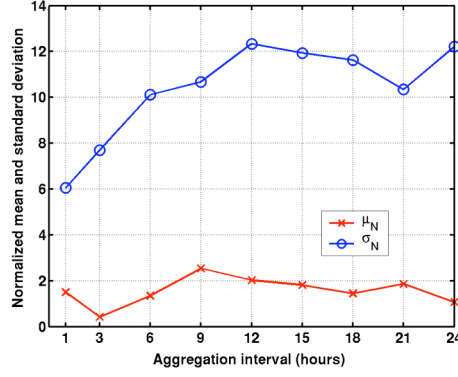
Fig. 7 Normalized mean and standard deviation of the modeling errors as a function of state transition matrix aggregation interval.

Mean aircraft counts, μ, mean of modeling errors, μ_e, normalized mean of the modeling errors, μ_N, standard deviation of the modeling errors, σ_e, and normalized standard deviation of the modeling errors, σ_N, for all the 23 Centers on 7 May 2003 with the state transition matrix aggregated over one-hour interval are summarized in Table 1. This table shows that the normalized mean of the errors is less than 2 for all the Centers. The normalized standard deviation is less than 9 for all the Centers except for Hawaii (ZHN), where it is 13. Note that mean traffic count in Honolulu Center (ZHN) is 19, which is small compared to other Centers.

Table 1 Summary of modeling error statistics for 23 Centers on 7 May 2003 with state transition matrix aggregation interval of one-hour

|	μ	μ_e	μ_N	σ_e	σ_N
ZAB	132	1.52	1.15	8.10	6.13
ZAU	181	1.88	1.04	11.72	6.48
ZBW	131	1.72	1.31	8.53	6.51
ZDV	145	1.04	0.71	7.86	5.42
ZDC	192	3.01	1.57	12.5	6.51
ZFW	153	1.35	0.88	9.46	6.18
ZHU	135	2.13	1.58	8.00	5.92
ZID	154	1.37	0.89	10.96	7.12
ZJX	163	2.03	1.24	8.12	4.98
ZKC	155	1.50	0.97	7.52	4.85
ZLA	168	1.78	1.06	9.88	5.88
ZLC	102	0.59	0.57	7.33	7.19
ZMA	84	1.48	1.76	5.85	6.97
ZME	129	1.09	0.84	10.74	8.32
ZMP	150	2.12	1.42	9.79	6.52
ZNY	121	1.32	1.09	7.90	6.53
ZOA	114	1.67	1.47	7.08	6.21
ZOB	167	1.41	0.85	10.58	6.33
ZSE	90	0.99	1.10	5.34	5.93
ZTL	175	2.62	1.50	10.58	6.05
ZAN	42	0.13	0.31	2.72	6.47
ZHN	19	0.31	1.61	2.47	13.02
INTL	838	4.72	0.56	20.17	2.41

These same statistics for the aggregation interval of 12 hours is summarized in Table 2. Comparing Table 2 to Table 1, it is easily seen that the mean and the standard deviation values for all the Centers with 12-hour aggregation interval are about twice as much as those with one-hour aggregation.

Table 2 Summary of modeling error statistics for 23 Centers on 7 May 2003 with state transition matrix aggregation interval of 12 hours

	μ	μ_e	μ_N	σ_e	σ_N
ZAB	132	2.17	1.64	17.24	13.06
ZAU	181	4.04	2.23	20.02	11.06
ZBW	131	3.59	2.74	13.27	10.13
ZDV	145	1.32	0.91	21.25	14.65
ZDC	192	4.54	2.37	22.37	11.65
ZFW	153	2.36	1.54	22.32	14.59
ZHU	135	3.21	2.38	16.10	11.93
ZID	154	2.80	1.82	15.23	9.89
ZJX	163	2.75	1.69	16.28	9.99
ZKC	155	2.13	1.38	19.41	12.52
ZLA	168	3.91	2.33	14.82	8.82
ZLC	102	0.81	0.79	15.98	15.67
ZMA	84	2.17	2.58	8.61	10.25
ZME	129	2.45	1.90	16.28	12.62
ZMP	150	4.59	3.06	17.91	11.94
ZNY	121	2.48	2.05	13.05	10.78
ZOA	114	3.54	3.11	11.52	10.11
ZOB	167	3.77	2.26	27.08	16.22
ZSE	90	2.10	2.34	9.90	11.00
ZTL	175	3.56	2.03	21.60	12.34
ZAN	42	0.75	1.78	9.35	22.25
ZHN	19	0.63	3.33	3.48	18.31
INTL	838	13.40	1.60	109.64	13.08

To determine the performance of LDSM when the state transition matrix is based on past data and departure rates are from current data, simulations were done using 6 May 2003 and 7 May 2003 data to predict 8 May 2003 flow (averaged over 24-hours). Although the fit is worse and there is a lag in the behavior (see Fig. 8), the fit is still sufficient to be able to predict Center overloads and permit planning for resource management purposes.
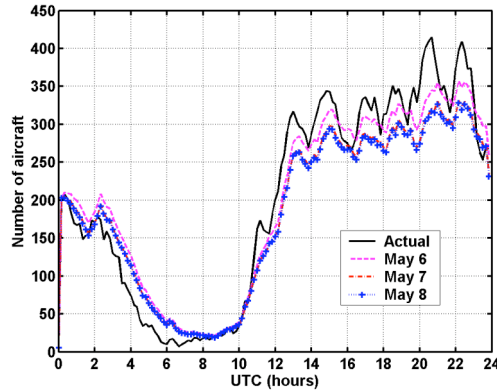
Fig. 8 Predicted ZTL aircraft counts with u updated every time step, but using a historical A from previous day (or from two days past) data aggregated over 24-hours.

It is thus clear that one can use a block-averaged version of **A** for modeling aggregate flow behavior.

V. Modeling of Departures from Airports

Discussions in the previous sections assumed that the departures within each Center are known a-priori. Since a significant amount of air traffic in high altitude airspace is airline traffic, the general traffic trends can be expected to remain unchanged from day-to-day because of fixed airline schedules. However, as seen in Fig. 5, the traffic count in the Centers does change from one day to the other because of convective weather and unavailability of some NAS resources. Weekday and weekend traffic trends also result in different traffic counts in the Centers. Many of these differences seen in the traffic patterns can be correctly modeled by LDSM constructed using historical data for days with similar characteristics.

As described earlier in Sec. II, LDSM allows for daily variations and other disturbances by using a deterministic component of the departures, **u**(k), and a stochastic component of the departures, **w**(k). The deterministic component could be the nominal estimate from all available data (e.g., historical departure data or proposed flight plans) while the stochastic component could be the uncertainty about the nominal estimate, which is also derived from the historical departure data. This approach implicitly accepts the fact that exact prior knowledge of the departures is difficult, therefore the traffic count computed using LDSM would have some uncertainty.

In Ref. 5, departures, $d(k)$, were modeled as samples from a single Poisson distribution. This approach can be generalized by modeling departures from several Poisson distributions over the 24-hour period. In order to take advantage of the many analysis techniques that are available for linear system models driven by Gaussian inputs, an alternative approach using the mean number of departures as the deterministic part of the input and variation about this mean as the stochastic part of the input (modeled as a Gaussian random variable) has been used in this paper. The mean number of departures can be computed from historical data as:

$$\mu_i(k) = \frac{\sum_{1 \leq L \leq D} d_{i_L}(k)}{D} \tag{7}$$

where D is the number of days and d_{i_L} is the number of departures from Center i on day L. Figure 9 shows the results of driving the LDSM with the mean departure counts computed over four days, 6 May 2003 through 9 May 2003, using Eq. (7). State transition matrices aggregated over one-hour periods were first obtained for each of the four days and then averaged (element-by-element means of the four transition matrices) to obtain the mean transition matrices valid for each one-hour period. These average transition matrices were then used in the LDSM and propagated forward with an integration step size of four minutes for obtaining the results shown in Fig. 9. This figure also shows actual traffic count, computed from recorded traffic data (not model-based), for each of the four days.

Examination of the traffic count time history, generated using mean departure counts, with respect to the actual traffic counts in Fig. 9 shows that the LDSM-generated counts approximate the mean traffic counts quite well. This figure also shows that the actual traffic count on any given day differs from the LDSM-generated traffic count. Therefore, uncertainty modeling is required to ensure that actual counts lie within the range established by these bounds. The mean traffic count prediction along with the uncertainty bounds about it can then be used for traffic flow management decisions.
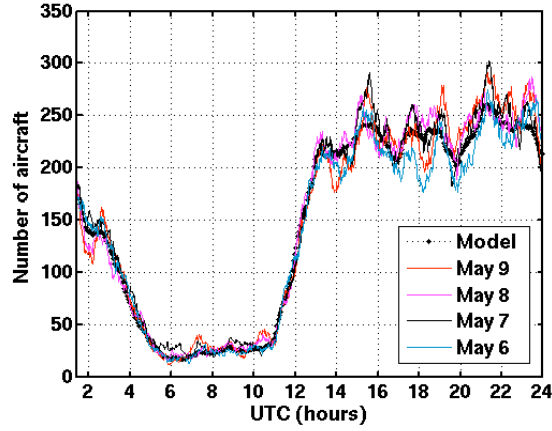
Fig. 9 Traffic count in ZFW calculated using mean departure counts and state transition matrix averaged over one-hour intervals, and actual traffic counts during four days in May.

In order to model the variation about the mean traffic count, the standard deviation of the departures can be computed from historical departure data as:

$$\sigma_i(k) = \sqrt{\frac{\sum_{1 \leq L \leq D} \left(d_{i_L}(k) - \mu_i(k) \right)^2}{D-1}} \tag{8}$$

The stochastic component of the input $\mathbf{w}(k)$ (see Eq. (2)) can be modeled with each component $w_i(k)$ as a random variable from the Normal distribution with the mean given by Eq. (7) and the standard deviation given by Eq. (8). Note that the distribution is assumed to be non-stationary and the mean and variance change with time k.

The normalized distribution of departures for three days, 6 May 2003 through 8 May 2003, is shown in Fig. 10. The departure distribution was obtained by first computing an average traffic count during each time interval (10-minutes) using the three days of data and then computing the difference of traffic counts for each day with respect to the average during each time interval in the 24-hour period. The difference counts for the three days were put together in a single dataset for creating the histogram. The resulting histogram was then normalized with respect to the area under the histogram to obtain the probability density function. This graph is shown with the 'x' symbol in Fig. 10.

The mean and the standard deviation values obtained from the dataset were used for fitting a Gaussian probability density function, which is marked by the 'o' symbol in Fig. 10. Although the Gaussian probability density function does not fit the actual probability density function perfectly, it is a reasonable approximation. It remains to be seen whether the approximation would improve if this analysis were repeated with several more days (for example, 30 days) of traffic data.
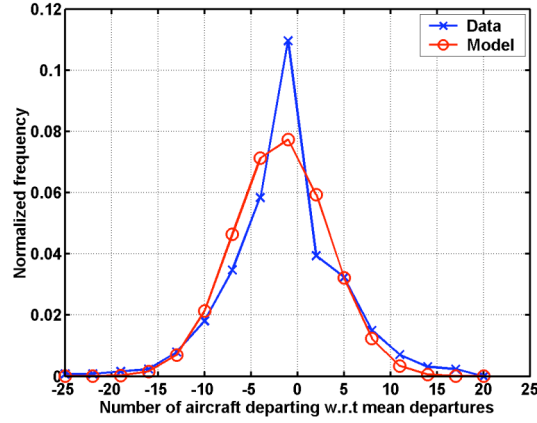
Fig. 10 Probability density function of Atlanta Center departures with respect to the mean number of departures and its approximation with a Gaussian model.

VI. Modeling Uncertainty Bounds

The process of computing uncertainty bounds about the LDSM-based nominal traffic count prediction is described in this section. The results presented in the previous sections (for example, see Fig. 9) were obtained using the deterministic part of the input, consisting of mean departure counts in the Centers. In this section, use of the stochastic part of the input along with the LDSM model in Eq. (2) in establishing uncertainty bounds is discussed.

Assuming $\mathbf{w}(k)$ is a vector of discrete time white noise sequences with covariance $\mathbf{Q}(k)$ in LDSM, given in Eq. (2), the uncertainty bounds can be obtained in terms of the state covariance matrix $\mathbf{P}(k)$ using the following recursive equation:[8]

$$\mathbf{P}(k+1) = \mathbf{A}(k)\mathbf{P}(k)\mathbf{A}^T(k) + \mathbf{C}(k)\mathbf{Q}(k)\mathbf{C}^T(k) \tag{9}$$

This equation is also used for state covariance propagation in the process update step of a Kalman Filter.[9] Propagation of the state covariance using Eq. (9) requires knowledge of the covariance matrix $\mathbf{Q}(k)$ and the initial state covariance matrix $\mathbf{P}(0)$. Starting with a null matrix of size 23×23, the diagonal terms of the $\mathbf{Q}(k)$ matrix are set to the variances of the numbers of takeoffs in the Centers.
Thus,

$$\begin{aligned} q_{i,j}(k) &= 0 \quad \forall \, i \neq j \\ &= \sigma_i^2(k) \quad \forall \, i = j \end{aligned} \tag{10}$$

$\sigma_i^2(k)$ is the variance in the number of takeoffs in the Centers computed using Eq. (8). To initiate the covariance propagation process, the state covariance matrix $\mathbf{P}(0)$ is initialized to a null matrix.

The time histories of average traffic counts for the Fort Worth Center, obtained using LDSM in Eq. (2), and 3σ bounds, obtained using the covariance propagation Eq. (9), are shown in Fig. 11. The average traffic count is shown via the square symbol in Fig. 11. This figure also shows the traffic count history in the Fort Worth Center on 1 October 2003, which is marked with the circle symbol. Observe from the figure that the 1 October 2003 traffic counts lie within the uncertainty bounds ($\mu \pm 3\sigma$) obtained using LDSM based on 6 May 2003 through 9 May 2003 data.
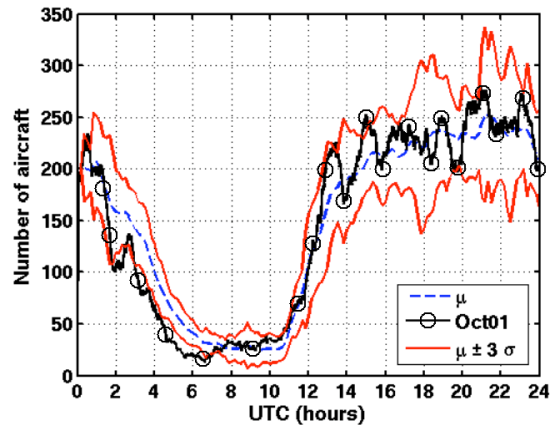
Fig. 11   Model-based average traffic counts and uncertainty bounds in the Fort Worth Center.

Although results have only been shown for the Fort Worth Center, the model-based traffic counts in other Centers were found to be of similar quality. These results illustrate that covariance propagation based on LDSM adequately describes the statistical variation seen in the day-to-day actual traffic flows.

VII.   Conclusions

We have described a class of linear time varying models to represent traffic flow for developing sound strategic traffic flow management decisions. The linear dynamic traffic flow system model with a slowly varying transition matrix and Gaussian departure representation was shown to adequately represent traffic behavior at the Center level. Furthermore, the method for computing uncertainty bounds around nominal traffic counts in the Centers was described. Numerical examples were presented using actual traffic data from four different days to demonstrate the model characteristics. The advantages of this class of models are: (1) Unlike trajectory-based models, these models are less susceptible to uncertainties in the system; (2) The model order is reduced by several orders of magnitude from 5000 aircraft trajectories to 23 states at any given time; and (3) A host of tools and techniques of modern system theory can be applied to this model because of its form. The capabilities of this class of models for strategic traffic flow management will be explored in the future.

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